Multi-Dimensional Cosmology and DSR-GUP

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Abstract

A multidimensional cosmology with FRW type metric having 4-dimensional space-time and d-dimensional Ricci-flat internal space is considered with a higher dimensional cosmological constant. The classical cosmology in commutative and DSR-GUP contexts is studied and the corresponding exact solutions for negative and positive cosmological constants are obtained. In the positive cosmological constant case, it is shown that unlike the commutative as well as GUP cases, in DSR-GUP case both scale factors of internal and external spaces after accelerating phase will inevitably experience decelerating phase leading simultaneously to a big crunch. This demarcation from GUP originates from the difference between the GUP and DSR-GUP algebras.

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1 Introduction

The Generalized Uncertainty Principal (GUP) is a generalization of Heisenberg Uncertainty Principal in the Planck scale where the gravitational effects on quantum gravity may be considerable. This idea, was firstly considered by Mead [1] and then implemented in the context of string theory as a candidate of quantum gravity as well as black hole physics with the prediction of a minimum measurable length [2, 3, 4, 5, 6, 7, 8]. Doubly Special Relativity (DSR) theory [9] as a possible ingredient of the flat space-time limit of the quantum theory of gravity proposed another modification on Heisenberg Uncertainty Principal [10]. Recently the authors in [11] considered these two modification as a limit of a single algebra (DSR-GUP).

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Nowadays, a large amount of interest has been focused on the effects of these modification on system in high energy physics [12]. In a recent paper [13], we have studied a multi-dimensional Cosmology with GUP and obtained the corresponding exact solutions for negative and positive cosmological constants. Especially, for positive cosmological constant, the solutions revealed late time accelerating behavior and internal space stabilized to the sub-Planck size, in good agreement with current observations. Motivated by the interest in DSR-GUP, in the present paper we are interested in studying the effects of DSR-GUP modifications on our multi dimensional cosmology and comparing its results with the GUP case[1].

In section 2, we introduce the notions of GUP and DSR-GUP as well as their corresponding algebras. In section 3, we briefly introduce our cosmological model. In section 4, we first obtain the commutative solutions and then find the DSR-GUP solutions. Finally, in conclusion, we compare the commutative, GUP and DSR-GUP solutions.

2 Generalized uncertainty principal

The simplest form of the GUP in a one dimensional system can be written as [8]

$$\delta x \delta p \geq \frac{\hbar}{2}\left(1+\beta L_{Pl}^2(\delta p)^2\right), \tag{1}$$

where $L_{Pl} \sim 10^{-35}m$ is the Planck length and β is a constant of order unity. The algebra corresponding to (1) can be written as [8]

$$[x_i, p_j] = i\{\delta_{ij} + \beta L_{Pl}^2(p^2\delta_{ij} + 2p_i p_j)\}, \tag{2}$$

which reduces to the ordinary one for $\beta \to 0$. Doubly Special Relativity theories, on the other hand, suggest that the planck scales similar to the light speed are observer independent scales. This is because different observers should not observe quantum gravity effects at different scales [9]. The algebra corresponding to DSR-GUP can be written as [10]

$$[x_i, p_j] = i\{\delta_{ij} - L_{Pl}|\overrightarrow{p}|\delta_{ij} + L_{Pl}^2 p_i p_j\}, \tag{3}$$

which reduces to the ordinary one for $L_{Pl} \to 0$. The authors in [11] showed that by assumption $[x_i, x_j] = 0 = [p_i, p_j]$, the two above algebra (2), (3) can be considered as a single algebra in phase space

$$[x_i, p_j] = i\left\{\delta_{ij} - \alpha L_{Pl}\left(p\delta_{ij} + \frac{p_i p_j}{p}\right) + \alpha^2 L_{Pl}^2(p^2\delta_{ij} + 3p_i p_j)\right\}, \tag{4}$$

where α is assumed to be of order unity and $p^2 = \sum_i p_i p_i$. By definition [11]

[1]Throughout the paper we will use the units $\hbar = G = c = 1$, where G is the gravitational constant and c is the velocity of light.

$$x_i = x_{i0}, \qquad p_i = p_{i0}(1 - \alpha L_{Pl} p_0 + 2\alpha^2 L_{Pl}^2 p_0^2), \tag{5}$$

the equation (4) can be satisfied, where x_{i0} and p_{i0} are the ordinary position and momentum with $[x_{i0}, p_{j0}] = i\delta_{ij}$ and $p_{j0} = -i\frac{\partial}{\partial x_{i0}}$. To distinguish between the linear and second order terms in Planck length, we rewrite equation (5) in a more general form

$$x_i = x_{i0}, \qquad p_i = p_{i0}(1 - \alpha L_{Pl} p_0 + \beta L_{Pl}^2 p_0^2). \tag{6}$$

Here, the coefficients α and β indicate the effect of linear and the second order terms in Planck length, respectively. So, setting $\alpha = 0$ and $\beta = 2\alpha^2$ gives back the ordinary GUP algebra (2) and the DSR-GUP algebra (4), respectively. Using (6), we can show that the p^2 term in any Hamiltonian can be derived as follows

$$p^2 = p_0^2 - 2\alpha L_{Pl} p_0^3 + (\alpha^2 + 2\beta) L_{Pl}^2 p_0^4. \tag{7}$$

3 The Cosmological Model

We consider a multi-dimensional cosmology in which the space-time is established by a FRW type metric with 4-dimensional space-time and a d-dimensional Ricci-flat internal space [14]

$$ds^2 = -dt^2 + \frac{R^2(t)}{(1 + \frac{k}{4} r^2)}(dr^2 + r^2 d\Omega^2) + a^2(t) g_{ij}^{(d)} dx^i dx^j, \tag{8}$$

where $R(t)$ and $a(t)$ are the scale factors of the external and internal spaces respectively, and $g_{ij}^{(d)}$ is the Ricci-flat metric of the internal space. The Ricci scalar is derived from the metric (8) [14]

$$\mathcal{R} = 6\left(\frac{\ddot{R}}{R} + \frac{k + \ddot{R}^2}{R^2}\right) + 2d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}}{a}\right)^2 + 6d\frac{\dot{a}\dot{R}}{aR}, \tag{9}$$

where a dot represents differentiation with respect to time t. The Einstein-Hilbert action with a $(3+d)$-dimensional cosmological constant Λ is written as

$$\mathcal{S} = \frac{1}{2k_{3+d}^2} \int_M d^{4+d}x \sqrt{-g}(\mathcal{R} - 2\Lambda) + \mathcal{S}_{YGH}, \tag{10}$$

where k_{3+d} is the $(3+d)$-dimensional gravitational constant and $\mathcal{S}_{YGH}$ is the York-Gibbons-Hawking boundary term. By substituting (9) in (10) and dimensional reduction we have

$$\mathcal{S} = -v_{3+d} \int dt \left\{ 6\dot{R}^2 \Phi R + 6\dot{R}\dot{\Phi} R^2 + \frac{d-1}{d}\frac{\dot{\Phi}^2}{\Phi} R^3 - 6k\Phi R + 2\Phi R^3 \Lambda \right\}, \tag{11}$$

where

$$\Phi = \left(\frac{a}{a_0}\right)^d, \tag{12}$$

and a_0 is the present time compactification scale of the internal space. We introduce the following change of variables provided $v_{3+d} = 1$ [13]

$$\Phi R^3 = \Upsilon^2 (x_1^2 - x_2^2), \tag{13}$$

$$\begin{aligned} \Phi^{\rho_+} R^{\sigma_-} &= \Upsilon (x_1 + x_2), \\ \Phi^{\rho_-} R^{\sigma_+} &= \Upsilon (x_1 - x_2). \end{aligned} \tag{14}$$

with

$$\begin{aligned} \rho_{\pm} &= \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{3}{d(d+2)}}, \\ \sigma_{\pm} &= \frac{3}{2} \pm \frac{1}{2}\sqrt{\frac{3d}{d+2}}, \\ \Upsilon &= \frac{1}{2}\sqrt{\frac{d+3}{d+2}}, \end{aligned} \tag{15}$$

where $R = R(x_1, x_2)$ and $\Phi = \Phi(x_1, x_2)$ are functions of new variables x_1, x_2. The above transformations with $k = 0$ result in the Lagrangian and Hamiltonian as follows

$$\mathcal{L} = (\dot{x_1}^2 - \dot{x_2}^2) + \frac{\Lambda}{2}\left(\frac{d+3}{d+2}\right)\left(x_1^2 - x_2^2\right), \tag{16}$$

$$\mathcal{H} = \left(\frac{p_1^2}{4} + \omega^2 x_1^2\right) - \left(\frac{p_2^2}{4} + \omega^2 x_2^2\right), \tag{17}$$

where

$$\omega^2 = -\frac{1}{2}\left(\frac{d+3}{d+2}\right)\Lambda. \tag{18}$$

4 Solutions

4.1 Commutative case

The dynamical variables defined in (14) and their conjugate momenta satisfy the following Poisson bracket algebra [14, 15]

$$\{x_\mu, p_\nu\}_P = \eta_{\mu\nu}, \tag{19}$$

where $\eta_{\mu\nu}$ is the two dimensional Minkowski metric. The equations of motion are obtained

$$\ddot{x_\mu} + \omega^2 x_\mu = 0. \tag{20}$$

For a negative cosmological constant ω^2 is positive and Eq.(20) describes the equations of motion for two ordinary uncoupled harmonic oscillators with solutions

$$x_\mu(t) = A_\mu e^{i\omega t} + B_\mu e^{-i\omega t}, \tag{21}$$

where A_μ and B_μ are constants of integration satisfying $A_\mu B^\mu = 0$ due to the Hamiltonian constraint ($\mathcal{H} = 0$). Using (12) and (14), the solutions for scale factors take the following forms

$$\begin{aligned} R(t) &= k_2[\sin(\omega t + \phi_1)]^{\frac{-\rho_-}{\rho_+\sigma_+ - \rho_-\sigma_-}}[\sin(\omega t + \phi_2)]^{\frac{\rho_+}{\rho_+\sigma_+ - \rho_-\sigma_-}}, \\ a(t) &= k_1[\sin(\omega t + \phi_1)]^{\frac{\sigma_+}{d(\rho_+\sigma_+ - \rho_-\sigma_-)}}[\sin(\omega t + \phi_2)]^{\frac{-\sigma_-}{d(\rho_+\sigma_+ - \rho_-\sigma_-)}}, \end{aligned} \tag{22}$$

where k_1 and k_2 are arbitrary constants and ϕ_1 and ϕ_2 are arbitrary phases. Imposing the Hamiltonian constraint leads to the following relation

$$\frac{4(d+2)}{d+3} k_1^d k_2^3 \cos(\phi_1 - \phi_2) = 0, \tag{23}$$

where because of k_1, $k_2 \neq 0$, it results in $\phi_1 - \phi_2 = \frac{\pi}{2}$. In what follows, we will investigate the behavior of a universe with one internal dimension ($D = 3+1$). By setting $\phi_1 = \frac{\pi}{2}$ and $\phi_2 = 0$, we obtain

$$\begin{aligned} R(t) &= k_2\sqrt{\sin(\omega t)}, \\ a(t) &= k_1\frac{\cos(\omega t)}{\sqrt{\sin(\omega t)}}. \end{aligned} \tag{24}$$

The Hubble and deceleration parameters for both $R(t)$ and $a(t)$ are calculated as

$$\begin{aligned} H_R(t) &= \frac{\dot{R}(t)}{R(t)} = \frac{\omega}{2}\cot(\omega t), \\ q_R(t) &= -\frac{R(t)\ddot{R}(t)}{\dot{R}^2(t)} = 1 + 2\tan^2(\omega t), \\ H_a(t) &= \frac{\dot{a}(t)}{a(t)} = -\frac{\omega}{2}(\cot(\omega t) + 2\tan(\omega t)), \\ q_a(t) &= -\frac{a(t)\ddot{a}(t)}{\dot{a}^2(t)} = -\frac{2\cos^2(\omega t)(5 + \cos(2\omega t))}{(-3 + \cos(2\omega t))^2}. \end{aligned} \tag{25}$$

Figure 1: Time evolution of the (squared) scale factors of universe with one extra dimension and negative cosmological constant. Solid, dashed-blue and dashed-green lines refer to the scale factors in commutative, DSR-GUP and GUP framework respectively. Left and right figures are the external and internal dimensions respectively.

The time evolution of $R^2(t)$ and $a^2(t)$ are depicted in Fig.1 (solid lines). According to this behavior, the universe begins from a big bang at $t = 0$, expands till $t = \frac{\pi}{2\omega}$ toward a maximum value, and starts contracting toward a big crunch at $t = \frac{\pi}{\omega}$.

Using the present value of Hubble constant, the age of universe becomes $t_{present} = \frac{1}{\omega}\cot^{-1}(\frac{2H_0}{\omega}) \approx \omega^{-1} \approx 10^{17}s$ which is in agreement with current observations. The present universe is also in tideway to get to maximum and minimum of $R^2(t)$ and $a^2(t)$, respectively, within $\Delta t \approx 0.57\omega^{-1}$.

We set the initial condition at planck time $R(t_{Pl}) = a(t_{Pl})$ and according to Fig.1, we see that during the whole time evolution of universe ($t_{Pl} \leq t \leq \frac{\pi}{\omega} - t_{Pl}$), the scale factor of internal space is contracted towards the sizes very smaller than $a(t_{Pl})$, and so can never exceed $a(t_{Pl})$. Moreover, considering

$$\begin{aligned} R(t_{Pl}) &= k_2\sqrt{\sin(\omega t_{Pl})}, \\ a(t_{Pl}) &= k_1\frac{\cos(\omega t_{Pl})}{\sqrt{\sin(\omega t_{Pl})}}, \end{aligned} \tag{26}$$

the above initial condition results in

$$\frac{k_2}{k_1} = 10^{61}, \tag{27}$$

by which we obtain the following ratio

$$\frac{R(t)}{a(t)} = 10^{61}\tan(\omega t). \tag{28}$$

If the present radius of external space be equal to the radius of observed universe $10^{28}cm$, then the present radius of internal space becomes about the Planck length ($10^{-33}cm$) and this justifies the non observability of the extra dimension.

For a positive cosmological constant ω^2 is negative, so by replacing ω^2 with $-\omega^2$ in Eq.(20) and using the Hamiltonian constraint the new solutions are obtained

$$\begin{aligned} R(t) &= k_2[\cosh(\omega t)]^{\frac{-\rho_-}{\rho_+\sigma_+-\rho_-\sigma_-}}[\sinh(\omega t)]^{\frac{\rho_+}{\rho_+\sigma_+-\rho_-\sigma_-}}, \quad (29) \\ a(t) &= k_1[\cosh(\omega t)]^{\frac{\sigma_+}{d(\rho_+\sigma_+-\rho_-\sigma_-)}}[\sinh(\omega t)]^{\frac{-\sigma_-}{d(\rho_+\sigma_+-\rho_-\sigma_-)}}, \end{aligned}$$

Figure 2: Time evolution of the scale factors of universe with one extra dimension and positive cosmological constant. Solid, dashed-blue and dashed-green lines refer to the scale factors in commutative, DSR-GUP and GUP framework respectively. Left and right figures are the external and internal dimensions respectively.

where for $d = 1$ we have

$$\begin{aligned} a(t) &= k_1 \frac{\cosh(\omega t)}{\sqrt{\sinh(\omega t)}}, \\ R(t) &= k_2 \sqrt{\sinh(\omega t)} \end{aligned} \tag{30}$$

$$\frac{R(t)}{a(t)} = 10^{61} \tanh(\omega t), \tag{31}$$

and

$$\begin{aligned} H_R(t) &= \frac{\dot{R}(t)}{R(t)} = \frac{\omega}{2} \coth(\omega t), \\ q_R(t) &= -\frac{R(t)\ddot{R}(t)}{\dot{R}^2(t)} = 1 - 2\tanh^2(\omega t), \\ H_a(t) &= \frac{\dot{a}(t)}{a(t)} = \frac{\omega}{2}(-\coth(\omega t) + 2\tanh(\omega t)), \\ q_a(t) &= -\frac{a(t)\ddot{a}(t)}{\dot{a}^2(t)} = -\frac{2\cosh^2(\omega t)(5 + \cosh(2\omega t))}{(-3 + \cosh(2\omega t))^2}. \end{aligned} \tag{32}$$

As in the case of negative cosmological constant, the magnitude of the radius of external to internal spaces is asymptotically ($t \to \infty$) about 10^{61}. As is seen in Fig. 2, $R(t)$ is an increasing function of time whereas $a(t)$ at first decrease with time till $t \simeq 0.88\omega^{-1}$ and then increase exponentially. If the age of universe is taken as $\omega^{-1} \simeq 10^{17}s$, then we find that at present time we are around the minimum point of $a(t)$ and that in the time interval $t_{Pl} \le t \le 141\omega^{-1}$, $a(t)$ can never exceeds $a(t_{Pl})$. This indicates that the internal scale factor remains very small, at least for 140 times of the present age of the universe.

The results obtained here with a positive cosmological constant are consistent with the current observations on the acceleration of the universe. Tho confirm this, we have depicted H_R, H_a and q_R, q_a in the figures 3 and 4 (see solid lines). Figure 3 shows that q_R becomes negative a little bit earlier than the present age of the universe namely $\omega t \sim 1$.

Figure 3: Left and right figures are respectively Hubble an deceleration parameters of internal space for universe with one extra dimension and positive cosmological constant. Solid , dashed-blue and dashed-green lines refer to the commutative, DSR-GUP and GUP framework respectively.

This means, the universe has started its acceleration recently. Fig.4 shows that q_a is always negative and has a minimum at the position where q_R becomes negative. The figures 3 and 4 indicate that at the beginning of time in both commutative and GUP cases, q_R is positive (R is decelerating) and q_a is negative (a is accelerating). With time evolution, q_R approaches the threshold of negative values (R is less decelerating) while q_a approaches to more negative values (a is highly accelerating). Once q_R enters the region of negative values (R is accelerating), q_a reaches its minimum (a stops its increasing acceleration). Finally, q_R becomes more negative (R is highly accelerating) whereas q_a goes to rather less negative values (a is slowly accelerating). It is interesting to note that the late time behavior of the universe is more considerable in the GUP case, where both R and a exhibit highly accelerating features.

4.2 DSR-GUP solutions

In this section, we aim to study this cosmological model in the DSR-GUP context to find effects of new terms in commutation relations on the time evolution of universe. The new terms in commutation relations can be considered in two view point: first order in Planck length due to DSR theory and second order term in Planck length due to GUP in string theory. Following equation (7) and (17) we write perturbed Hamiltonian as

$$\mathcal{H} = \frac{p_0^2}{2} - \alpha L_{Pl} p_0^3 + \frac{(\alpha^2+2\beta)}{2} L_{Pl}^2 p_0^4 + \omega^2(x_1^2 - x_2^2), \tag{33}$$

where $p_0^2 = \frac{p_{10}^2}{2} - \frac{p_{20}^2}{2}$ and $[x_{i0}, p_{j0}] = i\delta_{ij}$. Here, we want to investigate the classical version of DSR-GUP algebra. To do this, we must replace the quantum mechanical commutators with the classical poisson bracket as $[P, Q] \to i\{P, Q\}$. Using equation (19), the equations of motion can be written as

$$\begin{aligned} \dot{x}_\mu &= \{x_\mu, \mathcal{H}\}_P = \frac{1}{2}p_\mu - \frac{3}{2}\alpha L_{Pl} p_0 p_\mu + (\alpha^2+2\beta) L_{Pl}^2 p_0^2 p_\mu, \\ \dot{p}_\mu &= \{p_\mu, \mathcal{H}\}_P = -2\omega^2 x_\mu. \end{aligned} \tag{34}$$

Figure 4: Left and right figures are respectively Hubble an deceleration parameters of external space for universe with one extra dimension and positive cosmological constant. Solid , dashed-blue and dashed-green lines refer to the commutative, DSR-GUP and GUP framework respectively.

We see that deformed classical equations form a system of nonlinear coupled differential equation, so we need numerical solutions. Setting $\alpha = 0$ reduces the equations to GUP framework so we can see effect of the second order term in Planck length on the time evolution of universe. To investigate effect of the first order in Planck length due to DSR theory, we set $\beta = \alpha^2 = 0$.

In the negative cosmological constant framework, ω^2 is positive. Numerical solution of equations (34) shows that the deformed scale factors like commutative and GUP cases have periodic behavior. To compare with the results obtained in GUP [13], we have included the behaviour of the scale factors in GUP as well as commutative cases within the figures. As is seen in Fig.1, the time interval between big bang and big crunch in GUP case is shortened with respect to commutative one while in DSR-GUP case the time interval between big bang and big crunch is longer. In GUP case, the deformed scale factor of the internal space reaches it's minimum value sooner than commutative one while the deformed scale factor of the internal space in DSR-GUP case reaches later than commutative one. The deformed scale factor of the external space in GUP case reaches it's maximum sooner than commutative one and has larger value while in the DSR-GUP case, the deformed scale factor of the external space has smaller value and reaches to it later than commutative one.

Replacing ω^2 with $-\omega^2$ in Eq.(34), leads to the corresponding equations in the case of positive cosmological constant. Numerical analysis shows that (Fig.2) at early times the deformed scale factors of internal and the external spaces in both GUP and DSR-GUP cases behave like commutative one. At later times in the GUP framework, the expanding rate of deformed scale factors are bigger than commutative case while in the DSR-GUP case the deformed scale factors increase slower than commutative case. Moreover, at very late times after experiencing a maximum value, both scale factors decrease towards a big crunch simultaneously. Looking at the behaviour of deceleration parameter in figures 3 and 4 for internal and external scale factors, respectively, shows that unlike the commutative and GUP cases, in DSR-GUP case both scale factors after accelerating phases experience a decelerating phase at late times. This makes a remarkable difference between the DSR-GUP in one hand, and commutative together with GUP cases on the other hand. The

difference between GUP and DSR-GUP is due to a relative sign difference in the algebras (2) and (3) corresponding to GUP and DSR-GUP. This is interesting because the final fate of our multidimensional cosmology, being accelerated forever or decelerated towards a big crunch, is simply related to a relative sign difference in the quantum algebra corresponding to two different generalized uncertainty principles, GUP and DSR-GUP. Note also that the maximums of scale factors and the temporal location of big crunch in DSR-GUP case (Fig.2) depends on the Planck length: the more smaller Planck length, the more distant big crunch.

5 Conclusion

We have studied a multidimensional cosmology having FRW type metric with a 4-dimensional space-time sector and a d-dimensional Ricci-flat internal space subjected to a higher dimensional cosmological constant in the frameworks of commutative and DSR-GUP contexts. The corresponding exact solutions for negative and positive cosmological constants are obtained and compared with each other as well as GUP case. It is shown in DSR-GUP case that for positive cosmological constant, both scale factors of internal and external spaces after accelerating phase, unlike the commutative and GUP cases, will inevitably experience decelerating phase leading simultaneously to a big crunch. This unexpected behaviour originates from a negative sign in the DSR-GUP algebra.

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